

Math 111I  
Joseph C Foster  
Fall 2017  
Exam 1

Name: Solutions  
September 28th, 2017  
Time Limit: 75 minutes

This exam contains 7 pages (including this cover page) and 21 questions.  
The total number of points is 100. You have 75 minutes to complete the exam.

Read each question carefully. When specified, you must show **all necessary** work to receive full credit.

No calculator/phone/smartwatch allowed under any circumstances. Place these items in your bag, out of reach. Cheating of any kind will not be tolerated and will result in a grade of zero.

Question	Marks	Score	Question	Marks	Score
1	3		12	6	
2	3		13	6	
3	3		14	6	
4	3		15	6	
5	3		16	6	
6	3		17	6	
7	3		18	6	
8	3		19	7	
9	3		20	7	
10	3		21	8	
11	6		Total	100	

1. (3 marks) True or False:  $(A + B)^2 = A^2 + B^2$ .

A. True

B. False

2. (3 marks) True or False:  $\frac{7x + 2y}{x} = 7 + 2y$ .

A. True

B. False

3. (3 marks) True or False:  $x^2 - 4x - 21 = (x + 7)(x - 3)$ .

A. True

B. False

4. (3 marks) Fill in the blank: Given a quadratic equation  $ax^2 + bx + c = 0$ , the roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5. (3 marks) Fill in the blank: The *domain* of a function  $f(x)$  is all values of  $x$  for which  $f(x)$  is defined.

6. (3 marks) Fill in the blank:  $\sqrt{x - 5}$  is defined provided  $x - 5 \geq 0$ .

For questions 7-10, choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive 1 mark.

7. (3 marks) An example of an integer that is *not* a natural number is:

A. 0

C.  $-\frac{3}{2}$

B.  $\frac{1}{2}$

D.  $\sqrt{2}$

8. (3 marks) An example of an irrational number is:

A. 0

C.  $-\frac{3}{2}$

B.  $\frac{1}{2}$

D.  $\sqrt{2}$

9. (3 marks) How many roots does  $f(x) = 5x^2 - 17x - 600$  have?

A. 0

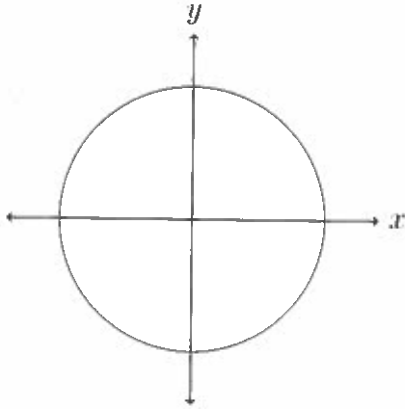
B. 1

$$17^2 - 4(5)(-600) \quad \text{C. 2}$$

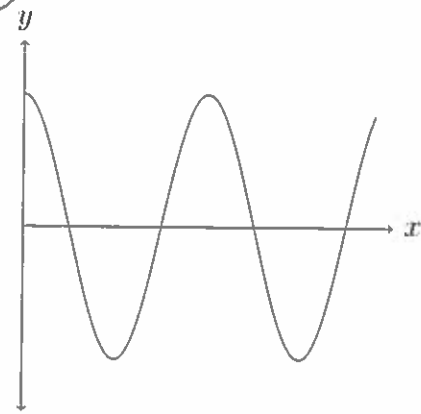
$$= 17^2 + 4(5)(600) > 0 \quad \text{D. 3}$$

10. (3 marks) Which of the following graphs represents a function:

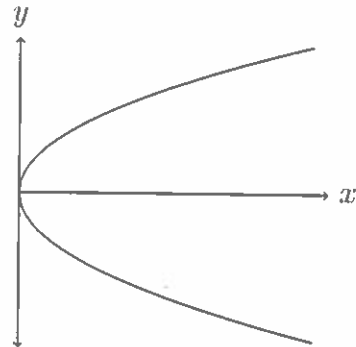
A.



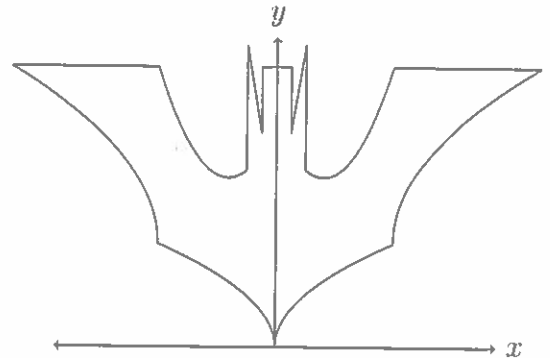
C.



B.



D.



11. (6 marks) Fill in the table below. You may assume that all letters represent positive numbers.

$a^m a^n = a^{m+n}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$a^{1/2} = \sqrt{a}$
$\frac{a^m}{a^n} = a^{m-n}$	$a^0 = 1$	$a^{1/n} = \sqrt[n]{a}$
$(a^m)^n = a^{mn}$	$a^{-1} = \frac{1}{a}$	$a^{m/n} = \sqrt[n]{a^m}$
$(ab)^n = a^n b^n$	$a^{-n} = \frac{1}{a^n}$	

For questions 12 – 21 show *all necessary* work to receive full credit. Circle or box your final answer.

12. (6 marks) Evaluate the arithmetic expression.

$$\begin{aligned} & \frac{5+7}{3} - 6[12 - (17 - 2 \cdot 3)] \\ & = \frac{12}{3} - 6[12 - (17 - 6)] \\ & = 4 - 6[12 - 11] \\ & = 4 - 6[1] \\ & = 4 - 6 \end{aligned} = \boxed{-2}$$

13. (6 marks) Expand the following expression using the distributive property.

$$(3q - 2qr - 5r)(-2ps)$$

$$\begin{aligned} & = 3q(-2ps) - 2qr(-2ps) - 5r(-2ps) \\ & = \boxed{-6pqs + 4pqrs + 10prs} \end{aligned}$$

14. (6 marks) Perform the indicated operation and simplify your answer.

$$\frac{y^2 + 2y - 3}{y^2 - 2y - 3} \div \frac{3 + y}{3 - y}$$

$$\begin{aligned} & = \frac{(y+3)(y-1)}{(y-3)(y+1)} \cdot \frac{(3-y)}{(3+y)} \\ & = \frac{-(y-1)(y-3)}{(y-3)(y+1)} \\ & = \frac{-(y-1)}{y+1} \end{aligned} = \boxed{\frac{1-y}{1+y}}$$

15. (6 marks) Perform the indicated operation and simplify your answer.

$$\begin{aligned} \frac{x}{(x+1)^2} - \frac{2}{x+1} &= \frac{x}{(x+1)^2} - \frac{2(x+1)}{(x+1)^2} \\ &= \frac{x - 2(x+1)}{(x+1)^2} \end{aligned}$$

$$= \boxed{\frac{x - 2x - 2}{(x+1)^2}}$$

16. (6 marks) Rationalise the denominator and simplify your answer.

$$\begin{aligned} \frac{2(x-y)}{\sqrt{x}-\sqrt{y}} &= \frac{2(x-y)(\sqrt{x}+\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} \\ &= \frac{2(x-y)(\sqrt{x}+\sqrt{y})}{x-y} \end{aligned}$$

$$= \boxed{2(\sqrt{x}+\sqrt{y})}$$

17. Find a linear model to represent the data for each of the following tables.

(a) (3 marks)

u	v
0	205
1	218
2	231
3	244

$$v = A + Bu \quad B = \frac{218 - 205}{1 - 0} = \frac{13}{1} = 13$$

$$205 = A$$

$$\boxed{v = 205 + 13u}$$

(b) (3 marks)

x	y
0	110
2	98
4	86
6	74

$$y = A + Bx \quad B = \frac{98 - 110}{2 - 0} = \frac{-12}{2} = -6$$

$$110 = A$$

$$\boxed{y = 110 - 6x}$$

18. (6 marks) Find *all* solutions to the inequality.

$$2(4 + 5y) \leq 12y - 6(1 - 3y)$$

$$\Rightarrow 8 + 10y \leq 12y - 6 + 18y$$

$$\Rightarrow 8 + 6 \leq 12y + 18y - 10y$$

$$\Rightarrow 14 \leq 20y$$

$$\Rightarrow \frac{14}{20} \leq y$$

$$\boxed{\frac{7}{10} \leq y}$$

19. (7 marks) Find *all* roots of the polynomial equation. (*Hint: simplify first.*)

$$(x + 1)(x^2 + 10x + 6) - (x + 1)5x = 0$$

$$\Rightarrow (x + 1)(x^2 + 10x + 6 - 5x) = 0$$

$$\Rightarrow (x + 1)(x^2 + 5x + 6) = 0$$

$$\Rightarrow (x + 1)(x + 2)(x + 3) = 0$$

$$\boxed{x = -1, -2, -3}$$

20. (a) (2 marks) Find the domain of the function  $f(x) = \sqrt{x+7}$

$$x+7 \geq 0 \Rightarrow \boxed{x \geq -7}$$

(b) (2 marks) Find the domain of the function  $g(x) = \sqrt{2-x}$

$$2-x \geq 0 \Rightarrow 2 \geq x$$

(c) (3 marks) Find the domain of the function  $h(x) = \frac{\sqrt{x+7}}{\sqrt{2-x}}$

$$\begin{aligned} x+7 \geq 0 &\Rightarrow x \geq -7 \\ 2-x > 0 &\Rightarrow 2 > x \end{aligned}$$

$$\boxed{-7 \leq x < 2} \quad \text{or} \quad \boxed{[-7, 2)}$$

21. In a certain country, income tax  $T$  is assessed based on a persons total income (in dollars). A person is charged as follows;

- If the persons income is \$10,000 or less, no tax is charged.
- If the person earns more than \$10,000 but no more than \$20,000, they are charged 8% of their total income.
- If the person earns more than \$20,000, they are charged the same <sup>8%</sup> ~~8%~~ on anything up to \$20,000 and a further 15% on anything over 20,000.

(a) (5 marks) Find a piecewise function  $T$  that represents the tax paid by a person earning  $x$  dollars.

$$0.08(20,000) = 1600$$

$$T(x) = \begin{cases} 0 & x \leq 10,000 \\ 0.08x & 10,000 < x \leq 20,000 \\ 1600 + 0.15(x-20,000) & 20,000 < x \end{cases}$$

(b) (3 marks) Find  $T(5,000)$ ,  $T(12,000)$  and  $T(25,000)$ .

$$\begin{aligned} T(5,000) &= 0 \\ T(12,000) &= 0.08(12,000) \\ T(25,000) &= 1600 + 0.15(5,000) \end{aligned}$$

